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Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Laminar Boundary Layers Subjected to High-Frequency Traveling-Wave Fluctuations"

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GREENBLATT and Damelin[1] recently presented an analysis of the flow in an incompressible, laminar boundary layer subjected to high-frequency, periodic freestream velocity fluctuations of the traveling-wave type. It is my purpose to point out that their analysis contains a fundamental error. This lies in the omission, from their equation for the high-frequency viscous sublayer, of the contribution to the unsteady pressure gradient arising from the wave convection. This term is of the order of the frequency ω and must be included in the high-frequency approximation, along with the contribution from the unsteady acceleration that is included in the Lighthill[2]–Lin[3] equation for the standing-wave case. Reference is made to the previous work on the traveling-wave case by Horton and Lam,[4] Lam,[5] and Evans.[6]

The analysis of Ref. 1 starts with the usual boundary-layer equations for unsteady, laminar, incompressible flow. Following Lin,[3] the external stream velocity U_e is taken to consist of a steady, mean component $\bar{U}$, plus a periodic fluctuation U_w:

$$U_e(x,t) = \bar{U}(x) + U_w(x,t) \tag{1}$$

The velocity components inside the boundary layer are similarly decomposed. For simplicity, we assume here that $\bar{U}$ is constant, which does not affect our basic argument. Then, in complex notation, U_w can be written as

$$U_w(x,t) = U_1 \exp\{i\omega(t - x/Q)\} \tag{2}$$

where Q is the wave convection speed and U_1 is the wave amplitude, here assumed small compared with $\bar{U}$. Putting $Q = \infty$ gives a standing wave.

The unsteady pressure gradient acting on the boundary layer is

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial U_e}{\partial t} + U_e\frac{\partial U_e}{\partial x}$$

$$\cong i\omega U_1(1 - \bar{U}/Q)\exp\{i\omega(t - x/Q)\} \tag{3}$$

and consists of two opposing terms, the first arising from the temporal rate of change of U_e and the second from the wave convection.[4–6] The second term is absent in the standing-wave case and was omitted by Greenblatt and Damelin.[1] However, when Q is of the order of $\bar{U}$, both terms are of similar magnitude, and so the high-frequency approximation to the momentum equation, Eq. (7b) of Ref. 1, should contain an additional term, to become

$$\frac{\partial u_w}{\partial t} = \frac{\partial U_w}{\partial t} + \underline{\bar{U}\frac{\partial U_w}{\partial x}} + \nu\frac{\partial^2 u_w}{\partial y^2} \tag{4}$$

Inclusion of the underlined term alters the nature of the solutions, especially when $Q < \bar{U}$.

Horton and Lam[4] carried out an asymptotic analysis of the high-frequency traveling-wave case, wherein an inner solution for the viscous sublayer was matched with an inviscid solution for the outer part of the boundary layer. Some relevant points are now briefly reviewed.

Harmonic forms, similar to Eq. (2), are adopted for the fluctuating components of velocity in the boundary layer, so that, for example,

$$u_w(x,y,t) = u_1(x,y)\exp\{i\omega(t - x/Q)\} \tag{5}$$

Assuming small amplitudes and linearizing, the momentum equation for the viscous sublayer takes the form

$$\frac{d^2\bar{u}_{1_s}}{d\eta_s^2} - 2i\bar{u}_{1_s} = -2i(1 - \bar{U}/Q) \tag{6}$$

where $\eta_s = y\sqrt{(w/2\nu)}$, $\bar{u}_{1_s} = u_{1_s}(\eta_s)/U_1$ and the subscript s denotes values in the sublayer.

For a standing wave, the coefficient on the right side of Eq. (6) equals unity, and the equation then becomes the Lighthill[2]–Lin[3] equation with the following well-known solution:

$$\bar{u}_{1_s}|_{Q=\infty} = 1 - \exp\{-(1+i)\eta_s\} \tag{7}$$

which satisfies the boundary conditions that $u_{1_s}(0) = 0$ and u_{1_s} is bounded for $\eta_s \to \infty$.

The solution of Eq. (6) for finite Q is evidently

$$\bar{u}_{1_s} = (1 - \bar{U}/Q)[1 - \exp\{-(1+i)\eta_s\}] \tag{8}$$

The effect of the omission by Greenblatt and Damelin[1] of the term underlined in Eq. (4) is equivalent to setting $(1 - \bar{U}/Q)$ to unity, and so their solution for u_1 (not explicitly given) is the same as the standing-wave solution. Equation (8) of Ref. 1 has been checked as being consistent with this, and so the absence of the additional term from Eq. (7b) of Ref. 1 is not simply a misprint. [There is another error in Eq. (7b) that obviously is a misprint, however.]

Some properties of the function $\bar{u}_{1_s}(\eta_s)$ are of note. For large η_s, the amplitude tends asymptotically to $(1 - \bar{U}/Q)$, whereas the phase tends either to zero if $Q > \bar{U}$ or to -180 deg if $Q < \bar{U}$. The phase angle at the wall is either $+45$ deg if $Q > \bar{U}$ or -135 deg if $Q < \bar{U}$.

The implications of the aforementioned on the outer, inviscid part of the solution and the composite inner and outer matching can now be discussed, bearing in mind that $\bar{u}_1 \to 1$ at the outer edge of the whole boundary layer.

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Standing Wave $Q \to \infty$

In this case the magnitude and phase of u_1 at the outer edge of the sublayer match the velocity at the outer edge of the boundary layer. Hence, the velocity fluctuations in the external stream are transmitted without change down to the outer edge of the sublayer. This is the (trivial) outer inviscid solution for this special case. The phase angle at the wall is +45 deg relative to the phase of the external velocity.

Wave Traveling with Finite Speed $Q > \bar{U}$

The magnitude of u_1 at the outer edge of the sublayer does not now match that at the outer edge of the boundary layer, although the phase does. The unsteady component of flow in the boundary layer outside the sublayer is governed at high frequencies by a first-order inviscid ordinary differential equation whose (purely real) coefficients contain functions derived from the Blasius solution for the mean flow.[4] This is the first-order outer equation in a matched asymptotic expansion. Numerical solutions for a range of values of $Q/\bar{U}$ are given in Ref. 4. There is again no change of phase through this outer region, and the value of $\bar{u}_1$ at its inner edge is equal to $(1 - \bar{U}/Q)$, asymptotically matching the sublayer solution. In the special case when $Q = \bar{U}$, the opposing contributions to the unsteady pressure gradient exactly cancel out, and the freestream oscillation is transmitted inwards through the boundary layer entirely by viscous diffusion. The high-frequency approximation is inapplicable.

Wave Traveling with Speed $Q < \bar{U}$

In this case neither the magnitude nor the phase of u_1 at the outer edge of the sublayer matches those quantities at the outer edge of the boundary layer. At the sublayer edge, u_1 is 180-deg out of phase with U_w, and the phase at the wall is −135 deg. The same inviscid equation as before applies outside the sublayer, but there is now a logarithmic singularity at the point in the outer layer where the velocity in the mean boundary layer is equal to Q. The phase angle is constant at −180 deg between the edge of the sublayer and this point and then gradually increases from −180 deg to zero between the singularity and the outer edge of the boundary layer. In real flow this singularity is smoothed out in a second viscous layer, termed the critical layer, an analysis of which is outlined in Ref. 4.

In conclusion it is observed that Lin's procedure for calculating the modification to the mean flow caused by the fluctuations, which was invoked by Greenblatt and Damelin, relies on the fact that for standing waves the sublayer solution and its continuation through the rest of the boundary layer do not depend on the mean flow. For traveling waves this independence does not exist, and therefore Lin's procedure would only be useful for small amplitudes at best.

Numerical solutions for high frequency confirm the structural features described earlier.[4,5]

References

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Comment on "Laminar Flow Past Three Closely Spaced Monodisperse Spheres or Nonevaporating Drops"

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Nomenclature

A = constant in Table 1
C_{Ds} = total drag coefficient of a single sphere
$C_{D,k}$ = total drag coefficient of sphere k
d = distance between two spheres, as ratio of sphere diameter
d_{12} = distance between spheres 1 and 2, as ratio of sphere diameter
d_{23} = distance between spheres 2 and 3, as ratio of sphere diameter
Re = Reynolds number
α_k = ratio of effective drag coefficient of sphere k to that of isolated sphere

I. Introduction

A PAPER by Ramachandran et al.[1] presents a computational study of the way in which fluid flow deviates from that past an isolated sphere when two other identical spheres are present along the axis of flow. They conclude that the drag coefficient depends upon the Reynolds number and the spacing of the spheres, this spacing being the dimensionless ratio of the intersphere distance to the sphere diameter. The extent of the drag reduction was calculated for intersphere distances from 2 to 6 sphere diameters, and for Reynolds numbers between 1 and 200. The authors used curve-fitting techniques to derive the following equations, which have been adopted by a number of other workers:

$$\alpha_1 = \frac{C_{D,1}}{C_{Ds}} = 1 - 0.096 Re^{0.2475} d_{12}^{-0.965} \exp\left(\frac{0.4764}{d_{23}}\right) \quad (1)$$

$$\alpha_2 = \frac{C_{D,2}}{C_{Ds}} = 1 - Re^{0.1593}\left(\frac{0.2932}{d_{12}^{0.4876}} + \frac{0.1341}{d_{23}^{0.4242}}\right) \quad (2)$$

$$\alpha_3 = \frac{C_{D,3}}{C_{Ds}} = 1 - 0.325(\ln Re + 1)^{0.603} \exp\left(\frac{-0.282}{d_{12}}\right) d_{23}^{-0.385} \quad (3)$$

They reported[1] that "The average curve-fitting errors, i.e., the deviations of $C_{D,k} \cdots$ from the computer generated values, range from 2.4 ($k = 1$) to 4.7% ($k = 3$). Maximum errors are always less than 9% and may occur at low Reynolds numbers ($Re < 20$)." This may be considered a good fit, and suggests that the modeling will be of use in important applications such as the study of dusts and sprays. On the other hand it does not justify the reporting of constants to four significant figures.

II. Sensitivity of the Equations

If we take values of $Re = 50$ and $d_{12} = d_{23} = 4$, it can be seen that the constants in Eqs. (1–3) do not greatly affect the overall result by being rounded. For example, $Re^{0.2475} = 2.633$ and $Re^{0.25} = 2.659$; $d^{-0.946} = 0.269$ and $d^{-1} = 0.250$; $\exp(0.4764/d) = 1.126$ and $\exp(0.5/d) = 1.133$. These are differences of 1, 7.6, and 0.6%.

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